PHYS 320 ANALYTICAL MECHANICS

Dr. Gregory W. Clark
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Quadratic Air Resistance: projectiles

$$v_{ter} \equiv \sqrt{\frac{mg}{c}} \qquad \tau \equiv \frac{m}{cv_o}$$

$$\begin{cases} m\dot{v}_x = -c\sqrt{v_x^2 + v_y^2}\; v_x \\ m\dot{v}_y = -mg - c\sqrt{v_x^2 + v_y^2}\; v_y \end{cases}$$

Coupled differential equations!

TRICKY to deal with.
Solve numerically!

Maple with numerical solutions to differential equations

SOLVING DIFFERENTIAL EQUATIONS WITH MAPLE NUMERICALLY:

GWClark

```
> restart; with(plots):g:=9.8;b:=0.1;m:=2;
> eq := m·diff(x(t), t, t) = -g;
> ic := x(0) = 19.6, D(x)(0) = 196;
> soln := dsolve({eq, ic}, x(t), type = numeric);
> odeplot(soln, [t, x(t)], 0..80, labels = [t, x], title = "position vs. time");
> odeplot(soln, [t, diff(x(t), t)], 0..80, labels = [t, v], color = blue, title = "velocity vs. time");
>
```

You have to give Maple some numbers for all the constants!

```
restart : with(plots) :
g := 9.8; m := 1; vo := 20; theta := 45;
```

9.8

1

20

45

Here, c is Taylor's quadratic drag of F[quad]=-cv^2.

```
c := 0.01;
```

0.01

```
ay := diff(y(t), t, t) =-m·g - c·diff(y(t), t)·sqrt(diff(x(t), t)^2 + diff(y(t), t)^2);
```

$$\frac{d^2}{dt^2} y(t) = -9.8 - 0.01 \left(\frac{d}{dt} y(t)\right) \sqrt{\left(\frac{d}{dt} x(t)\right)^2 + \left(\frac{d}{dt} y(t)\right)^2}$$

```
ax := diff(x(t), t, t) =-c·diff(x(t), t)·sqrt(diff(x(t), t)^2 + diff(y(t), t)^2)
```

$$\frac{d^2}{dt^2} x(t) = -0.01 \left(\frac{d}{dt} x(t)\right) \sqrt{\left(\frac{d}{dt} x(t)\right)^2 + \left(\frac{d}{dt} y(t)\right)^2}$$

```
IC := D(y)(0) = vo·sin(theta·Pi/180), D(x)(0) = vo·cos(theta·Pi/180), y(0) = 0, x(0) = 0;
```

$$D(y)(0) = 10\sqrt{2}, D(x)(0) = 10\sqrt{2}, y(0) = 0, x(0) = 0$$

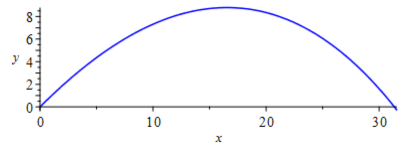
```
odesys := [ay, ax, IC];
```

$$\left[\frac{d^2}{dt^2} y(t) = -9.8 - 0.01 \left(\frac{d}{dt} y(t)\right) \sqrt{\left(\frac{d}{dt} x(t)\right)^2 + \left(\frac{d}{dt} y(t)\right)^2}, \frac{d^2}{dt^2} x(t) = -0.01 \left(\frac{d}{dt} x(t)\right) \sqrt{\left(\frac{d}{dt} x(t)\right)^2 + \left(\frac{d}{dt} y(t)\right)^2}, D(y)(0) = 10\sqrt{2}, D(x)(0) = 10\sqrt{2}, y(0) = 0, x(0) = 0\right]$$

```
dsol := dsolve(odesys, numeric);
```

proc(x_rkf45) ... **end proc**

```
P1 := odeplot(dsol, [x(t), y(t)], 0..2.7, color = blue);
```



Drag: linear vs. quadratic

- Linear $\vec{f}_{lin} = -b\,\vec{v}$

$v_{ter} \equiv \frac{mg}{b}$ $\tau \equiv \frac{m}{b}$

$b = \beta D = 3\pi\eta D$ *(D = Stokes diameter; η = viscosity)*

$\beta \approx 1.6x10^{-4}\,Ns/m^2$ *for sphere in air at STP*

- Quadratic $\vec{f}_{quad} = -c\,v\,\vec{v}$

$v_{ter} \equiv \sqrt{\frac{mg}{c}}$ $\tau \equiv \frac{m}{cv_o}$

$c = \gamma D^2 = \frac{1}{2}\rho C_d D^2$ *(D = Stokes diameter; ρ = fluid density; C_d = drag coefficient)*

$\gamma \approx 0.25Ns^2/m^4$ *for sphere in air at STP*

The Simple Harmonic Oscillator

- Differential equation:

$$m\frac{d^2x}{dt^2} + kx = 0 \qquad \omega_o \equiv \sqrt{\frac{k}{m}} = \text{ natural frequency}$$

- Solutions:

$$x(t) = C_1e^{+i\omega_o t} + C_2e^{-i\omega_o t} = B_1\cos(\omega_o t) + B_2\sin(\omega_o t) = A\cos(\omega_o t - \delta)$$

A test for linear independence:

- Two functions *f(x)* and *g(x)* are linearly independent when

  a f(x) + b g(x) = 0

  iff a = b = 0.

- The Wronskian determinant: If the Wronskian does not vanish, the functions that make up the Wronskian are linearly independent

$$W(x) \equiv \begin{vmatrix} f_1(x) & f_2(x) & \dots & f_n(x) \\ f_1'(x) & f_2'(x) & \cdots & f_n'(x) \\ \vdots & \vdots & \ddots & \vdots \\ f_1^{(n-1)}(x) & f_2^{(n-1)}(x) & \cdots & f_n^{(n-1)}(x) \end{vmatrix}$$

here, primes note derivatives with respect to *x*